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Financial Exclusion and Inflation Costs

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Keywords: Financial Exclusion; Inflation Costs; Costly Credit
JEL: D53; E31; E51; G23

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1 Introduction

Financial exclusion affects both developed and developing countries. The European Commission defines financial exclusion as "not being able to use financial services in the mainstream market that are appropriate to people's needs and enables them to lead a normal life in the society in which they belong" (European Commission, 2008). Individuals fitting this definition are called "unbanked". According to Fondeville et al. (2010), on average, 11.6% of EU households had no bank account. However, this number varies substantially across EU countries. In Bulgaria, Greece, and Romania, more than 70% of households lack a bank account. In the US, the FDIC *National Survey of Unbanked and Underbanked Households* reveals that US unbanked households vary between 7% and 8.2%.[1]

For most individuals being unbanked is a choice rather than a constraint. The FDIC Survey in 2015 reports that 57.4% of US households indicate that they do not have access to bank services because they "do not believe they have enough money to keep in an account" (FDIC, 2015). Others indicate that they do not need or want an account. Less than 2% of households say they do not have a bank account for a reason other than "no need/prefer dealing with money". For more complex bank services such as access to credit, lack of access is higher. In the EU, 40% of households have no credit card, overdraft facilities, or outstanding loans (excluding mortgages). In Bulgaria and Romania, about 70% of households have no credit services.

Comparing groups of developing countries, Latin America and the Caribbean have higher access rates while Africa and the developing countries of Eastern Europe and Central Asia have lower access rates. In Brazil, for example, the Financial Inclusiveness Report (Central Bank of Brazil, 2011) indicates that 49% of adults have no deposit accounts in formal financial intermediaries. Honahan (2008) estimates an indicator for the fraction of the adult population using formal financial intermediaries, whether through deposit accounts or by borrowing. For Russia, India and China, the numbers in Honahan (2008) are 31%, 52%, and 58%, respectively. Overall, using this indicator, Honahan finds a positive correlation between access to financial services and GDP.

Given the fact that most unbanked households have low income and that the rate of unbanked is high in developing countries, this paper studies an economy with banked and unbanked

[1]See `www.economicinclusion.gov/surveys`. The percentage of unbanked and FDIC Survey year are: 7.6% (2009), 8.2% (2011), 7.7% (2013), and 7% (2015).

individuals in order to understand the effect of inflation on financially excluded households. As argued by Erosa and Ventura (2002), the burden of inflation is not evenly distributed among households due to heterogeneous portfolio holdings and transaction patterns. The goal of the paper is to compute inflation costs for individuals with access to credit and individuals without access to assess differences in the burden of inflation.

This paper studies a pure exchange model based on Lucas and Stokey (1983) and a model with production and costly credit along the lines of Gillman (1993), Dotsey and Ireland (1996) and Erosa and Ventura (2002) to compute the costs of inflation on unbanked households. It defines the cost of inflation rate π as the share of consumption the household would need to receive to be indifferent between inflation rate π and a baseline inflation rate. The paper's main finding is that there is a substantial asymmetry in welfare costs between individuals who have access to financial services and those that lack access. Households with no access to the financial sector have a welfare cost that is 13 times greater than the cost for households with access.

In seminal papers Lucas (1994, 2000) defines the welfare costs of inflation as the percentage of income needed to leave a household indifferent between a nominal interest rate r and zero. Lucas estimates an increase in welfare equivalent to an increase in real income of 0.8% due to a reduction in interest rates from 14% to 3%. Moreover, an interest rate decline from 3% to 0% would lead to an increase in welfare equivalent to an increase in income between 0.1% and 0.9%, depending on the functional form of the demand for money. In contrast, Imrohoroglu (1992) used Bailey (1956)'s framework, which defines the welfare cost of inflation as the triangle below the demand curve for money. Her results show that the cost of inflation is 0.41% of GNP for an inflation rate of 10% and 0.14% of GNP for a 5% inflation rate. The main reason for the difference is that Imrohoroglu's economy has incomplete markets where money is held for precautionary savings, while in the present economy the friction is transaction costs.

Gillman (1993) estimates the costs of inflation in an economy where it is costly to acquire credit. In this setup, inflation costs are substantially higher than in an economy with costless credit. His findings show that the welfare cost of a 10% inflation equals 2.19% of the economy's income. In order to include costs of capital accumulation in the analysis, Dotsey and Ireland (1996) use a costly credit approach in a general equilibrium model. They find a cost of 0.41% of output for a 4% inflation rate. In a similar economy where one type of agent has college education, and the other type has no college education, Erosa and Ventura (2002) find that the welfare cost of inflation for the college-educated is approximately half of the welfare cost for those without college education. It stresses the regressive feature of an inflation tax.

In contrast to the vast literature on the welfare costs of inflation, assessing the effects of financial inclusion is a relatively new endeavor, especially in a general equilibrium model. For example, Besley et al. (2018) calibrate a general equilibrium model with ex-ante moral hazard and limited liability. The model introduces a financial inclusion parameter, which denotes the fraction of individuals with access to financial markets. As in Jeong and Townsend (2007), the parameter accounts for the fact that some agents face prohibitively high transaction costs. The main finding in the paper is that financial inclusion is more critical quantitatively than contract frictions, and remedying the problem can lead to significant wage increases.

Also recently, Liu (2018) constructs a general equilibrium model with heterogeneous agents and introduces financial exclusion due to minimum deposit size requirements imposed by banks. The deposit size restriction causes some individuals to become unbanked. The paper shows that appropriately designed microsavings programs can improve welfare. Neither Besley et al. (2018) or Liu (2018) consider the link between inflation and financial inclusion. T paper fills this gap.

The paper proceeds as follows. Section 2 presents data on the level of financial exclusion across countries. Section 3 contains a simple endowment model based on Lucas and Stokey (1983), where financially excluded agents differ in their ability to buy "credit goods". Section 4 contains a general equilibrium model with features along the lines of Gillman (1993), Dotsey and Ireland (1996) and Erosa and Ventura (2002). Section 5 presents concluding remarks.

2 Financial Exclusion across Countries

2.1 Reasons to be unbanked

The Global Financial Inclusion Database (Findex) in 2017 indicates that 49% of the world population did not have an account at a financial institution in 2011 (Demirguc-Kunt et al., 2018). Access grew to 67% in 2017, but the percentage of unbanked in low-income countries remains high. The Global Financial Development Database (GFDD) divides countries by their level of income. Using this partition, it becomes evident that the level of financial inclusion is increasing

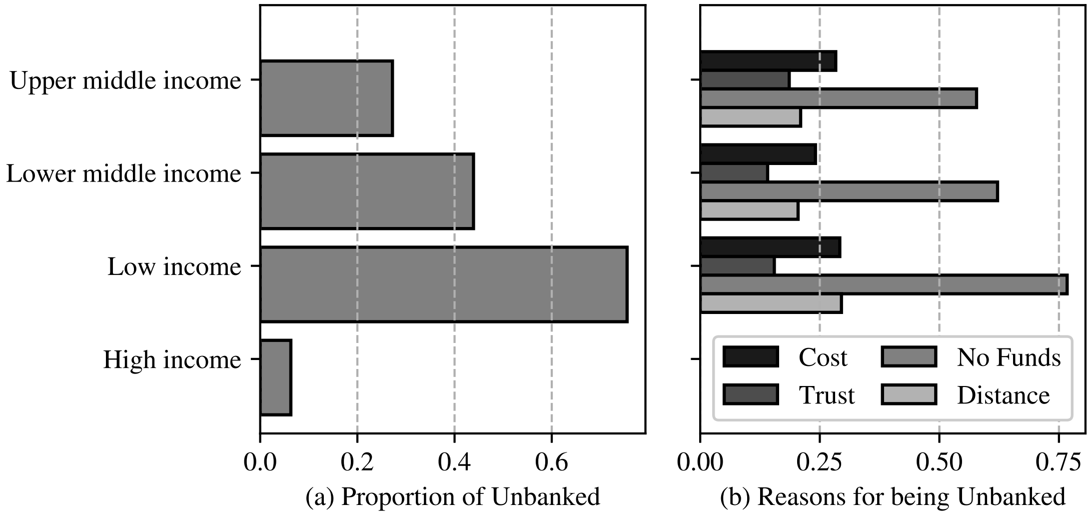


Figure 1: Proportion of unbanked agents and reasons for being unbanked

Note: Proportion of unbanked agents and reason for being unbanked are measured by the Findex Database 2017. The country groups are defined by the GFDD.

with income. In the 2017 Database, only 6.3% of individuals in high-income countries lacked financial services, while 75.5% of individuals were financially excluded in low-income countries. Upper-middle-income countries and lower-middle-income countries had 27.2% and 43.9% of their population financially excluded, respectively. Fig. 1a summarizes this data.

The Findex database is a household level survey that covers more than 155,000 adults in 148 countries. One of the topics the survey investigates is why some individuals do not own any formal financial services. According to the survey, 23.7% of unbanked individuals in low-income countries report that one of the reasons they did not have a bank account was because financial institutions were too far away (distance). Looking at the data by income group, we observe the same pattern as before: infrastructure problems seem to affect more harshly low-income countries. However, the variation among income groups is not significant. In lower-middle-income countries, 11.3% of the unbanked claim that distance is one of the reasons for financial exclusion, and in the upper-middle-income countries, only 10% of unbanked report this problem.

Guiso et al. (2004) notes that trust may also play an important role in the development of financial systems. In low-income countries, 15.5% of the respondents argue that lack of trust is one of the reasons for being financially excluded. This value is lower than the values for infrastructure problems (distance), but this reason seems to be more important in upper-middle-income countries where 18.6% of unbanked individuals answer 'yes' to this question, the highest percentage among the groups. Lower-middle-income countries report 14%.

Lastly, we analyze the economic reasons for being unbanked. Like any other service, we expect financial services to follow the law of supply and demand. Therefore, an apparent reason for financial exclusion is the cost to acquire those services. Cost is an important reason, mainly for low-income countries, although Fig. 1b shows that cost does not change much among groups. 28.3% report this answer in upper-middle-income countries, while 29.2% report cost as a reason in low-income countries.

The second economic reason for being unbanked is simply the lack of demand for the service. In this case, the question is whether an individual did not have enough funds, and this led to financial exclusion. Fig. 1b shows that 'no funds' is the most important reason. Moreover, this percentage is decreasing in the level of income, with 76.7% of the unbanked in low-income countries reporting that a lack of enough money is a reason for being unbanked. The percentage is 57.7% for upper-middle-income countries.

2.2 Credit Constraints and Informal Credit

The scenario gets worse when we turn the analysis to more complex services, like credit. According to the Findex 2017, only 18.4% of the respondents have a credit card. As for the unbanked, access to credit is increasing with the income of a country. In low-income countries, only 2% of individuals have a credit card, while in lower-middle-income countries, the number rises to 3%. We observe a higher increase for the case of upper-middle-income countries where 19.5% of the population do not have a credit card and even higher rates for high-income countries with 54.9%. Fig. 2a summarizes these results.

Unfortunately, the survey does not ask why an individual does not have a credit card. One reason is access to other forms of credit, and Findex provides information on this. The questions in the survey regarding "other kinds of borrowing" range from store credit to loans from friends or family. Fig. 2b shows that borrowing using store credit is more common in high-income countries. In 2017, 12.5% of the respondents acquired credit through stores in high-income countries.

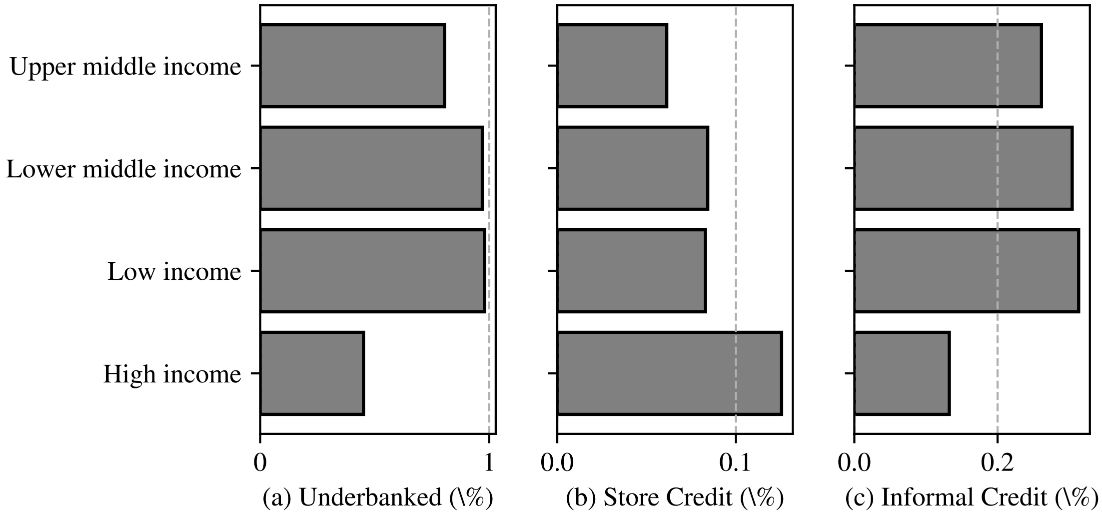


Figure 2: Underbanked and Credit Alternatives

Note: Data from the Findex database 2017. The countries groups are defined by the GFDD.

This number was 8.2% in low-income countries and only 6.1% in upper-middle-income countries. However, Fig. 2c shows that the pattern changes for informal credit. Lower-income countries had 31.3% of respondents acquiring credit from family or friends. This number is only 13.3% for high-income countries. The data suggest that individuals in poor and developing countries find a way around the lack of credit cards through informal credit.

3 A Pure Exchange Economy

The data in the previous section shows that financial exclusion is pervasive, especially in developing countries. In order to better understand the effects of financial exclusion, this section constructs an endowment economy model to evaluate the cost of inflation in two types of households: Banked and Underbanked. The model in this section augments the Lucas and Stokey (1983) pure exchange economy to include Banked and Underbanked households (i.e., households with and without financial access). There are two different agents, H and L, endowed with good w_t, which equals 1 in every period.

The only distinction between agents H and L is their ability to obtain credit. Agent H draws utility from consumption of two kinds of goods: c_{1t}, the "cash good", must be bought with fiat currency, and c_{2t}, the "credit good", can be bought with credit. Agent-L is not able to acquire credit. In other words, agent-L is financially excluded, and therefore she draws utility only from c_{1t}. The unbanked can borrow and lend in an informal market, which is consistent with the facts in the previous section.

The informal market is risk-free and pays the same interest as the formal market.[2] Therefore, both types of agents can hold bonds and money (M_t).

The date-0 budget constraint is given by

$$\sum_{t=0}^{\infty} q_t[c^j_{1t} + c^j_{2t} + (1+\pi_t)m^j_{t+1}] \leqslant \sum_{t=0}^{\infty} q_t(w_t + m^j_t), \tag{1}$$

for all $j = H, L$ where $m^j_t \equiv M^j_t/p_t$ is the real money balances, $(1+\pi_t) \equiv p_{t+1}/p_t$ is the inflation rate, $q_t \equiv \prod_s^t \frac{1}{1+r_s}$ is the Arrow-Debreu price and the constraint $c^L_{1t} = 0$ applies.

There is a standard cash-in-advance constraint: for all $j = H, L$,

$$c^j_{1t} \leqslant m^j_t, \tag{2}$$

Therefore, household j maximizes utility

$$\sum_{t=0}^{\infty} \beta^t u(c^j_{1t}, c^j_{2t}), \tag{3}$$

subject to Eqs. (1) and (2), transversality conditions for money and bonds and non-negativity constraints for consumption and money holding. The amount of money in the initial period is provided by the government.

The government produces an amount of unproductive bureaucratic goods G_t which is financed by seignorage such that the government budget constraint is

$$G_t = \frac{m^s_t - m^s_{t-1}}{p_t} = (1+g_{t-1})\frac{m^s_{t-1}}{p_t}, \tag{4}$$

where m^s_t is the supply of money in period t and g_t is the growth rate of money. The government chooses a money growth sequence $\{g_t\}_{t=0}^{\infty}$ and the initial amount of money m^s_0.

[2] Informal finance corresponds to funds from friends and family, consistent with Findex data discussed in section 2.

3.1 Equilibrium

The equilibrium must satisfy the following market clearing conditions:

$$\begin{aligned} &\lambda^H(c_{1t}^H + c_{2t}^H) + \lambda^L c_{1t}^L = w_t \\ &\lambda^H m_t^H + \lambda^L m_t^L = m_t^s \\ &\lambda^H b_{t+1}^H + \lambda^L b_{t+1}^L = 0 \end{aligned} \tag{5}$$

where λ^j is the number of j-type agents such that $\lambda^H + \lambda^L = 1$. Then, the competitive equilibrium for the model economy is defined as follows:

Definition 1. *A competitive equilibrium is a sequence of prices* $\{r_t, p_t\}_{t=0}^{\infty}$*, a sequence of government policies* $\{m_0^s, \{g_t\}_{t=0}^{\infty}\}$*, a sequence of consumer choices* $\{c_{1t}^j, c_{2t}^j, m_t^j, b_{t+1}^j\}_{t=0}^{\infty}$ *for all* $j = H, L$*, such that:*

- *Given prices and government policies, the consumers maximize Eq. (3) subject to Eqs. (1) and (2); and*
- *The market clearing conditions in Eq. (5) hold.*

To find the equilibrium, one can solve a system of equations composed of the market-clearing conditions in Eq. (5), the first-order conditions, budget constraints, and cash-in-advance constraints for both representative agents, and the government budget constraint.

Let λ be the Lagrange multiplier on the budget constraint and ϕ_t be the multiplier on the cash-in-advance constraint. From the households' problem, the first order conditions give:

$$\phi_t = \lambda q_t \left(1 - \frac{p_t}{p_{t+1}} \frac{1}{1 + r_{t+1}}\right). \tag{6}$$

The cash-in-advance constraint binds if and only if the return from bonds is bigger than the rate of return on money, which implies a positive net nominal interest rate.

The money demands follow from the cash in advance constraints at equality. Moreover, the relationship between the consumption of "cash goods" and "credit goods" is given by

$$\frac{c_{2t}}{c_{1t}} = 1 + \frac{i}{1+i}, \tag{7}$$

where $(1+i) = \frac{p_{t+1}}{p_t}(1+r_{t+1})$. Note that the consumption of "credit goods" relative to "cash goods" increases when inflation increases. For higher inflation rates type H households hold less money, which implies the consumption of "cash goods" falls. They increase the demand for "credit goods". Type L households lack access to credit by design and purchase only cash goods.

3.2 Quantitative analysis

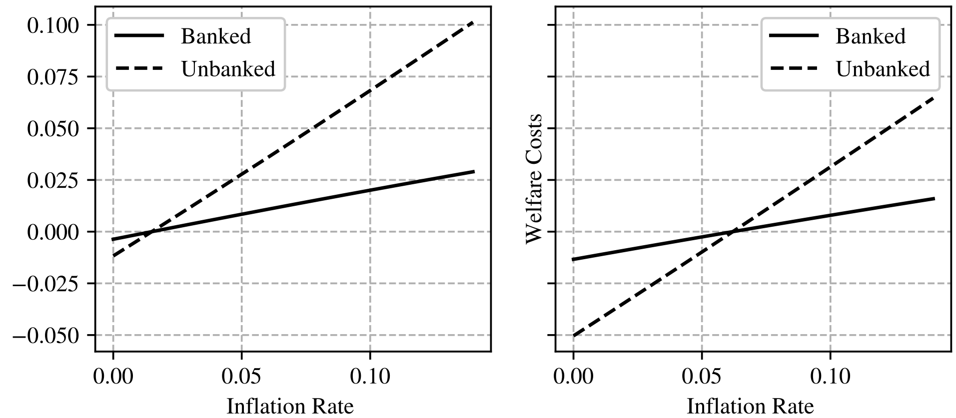
To proceed with the quantitative analysis, one needs to choose functional forms for preferences. I follow Gillman (1993) and Dotsey and Ireland (1996):

$$u(c_{1t}^L) = \ln c_{1t}^L \quad \text{and} \quad u(c_{1t}^H, c_{2t}^H) = \ln c_{1t}^H + \ln c_{2t}^H.$$

As in Lucas (1994, 2000), the consumption required by the agent to reach the same utility level as in the original case after a rise in inflation defines the welfare cost of inflation. The first case is the baseline economy calibrated for two types of countries, developed and developing countries. There are only two parameters to be chosen: the discount factor β, and the proportion of type-H agents N^H.

I use the steady state condition $(1+r) = \frac{1}{\beta}$ to determine the discount factor. The average real interest rate from the World Bank for the US is 1.7% and 4% for China over the period 2013-2015, which yields $\beta = 0.98$ and $\beta = 0.96$ for the discount factors of the developed and developing economies, respectively. The data for the proportion of banked households is in subsection 2.2, where the data for the developed economy comes from the high-income OECD countries, and the upper-middle-income countries present the data for the developing economy. Finally, baseline inflation is 1.5% for developed countries and 4% for developing countries. The inflation rate comes from The World Bank for Brazil and the US over the same period, representing developing and developed countries, respectively.

Fig. 3a presents the welfare costs of inflation for different inflation rates relative to the baseline economy for developed countries. Note that each type would be willing to give up consumption to reduce inflation. In the developed economy, a type-H agent would give up 0.36% of consumption to reduce inflation from 1.5% to zero. Because inflation is small, there is a minimal welfare gain from reducing inflation to zero. However, the cost for the financially excluded is



(a) Developed Countries (b) Developing Countries

Figure 3: Inflation costs in a pure exchange economy

much higher than for individuals with access to credit. In the developed economy, a type-L agent would be willing to give up 1.1% of consumption to live in a zero inflation economy. The welfare cost for the financially excluded is small, but approximately three times as much as the welfare cost for the type-H individuals.

Fig. 3b presents the welfare costs of inflation for different inflation rates relative to the baseline economy for developing countries. In the developing economy, inflation in the baseline economy is higher (6.2%), and the welfare cost is higher compared to the developed economy. For the type-H, the amount of consumption the agent would give up to have zero inflation is 1.3%. The type-L would give up about four times as much consumption for a zero inflation rate (5%).

The results in Fig. 3a and Fig. 3b show that the costs of inflation are relatively low, as in other papers in this literature, e.g., (Lucas, 2000; Cavalcanti and Villamil, 2003). However, the welfare costs are significantly higher for financially excluded individuals and in countries that are less developed. Keep in mind that the costs assessed in this model come solely from the fact that financially excluded agents cannot acquire credit for purchases in a given period; however, they can smooth consumption through risk-free bonds. Note that this model evaluates only this dimension of the impact of inflation on unbanked and underbanked households (i.e., financial exclusion).

4 A General Equilibrium Model with Costly Credit

Although the pure exchange model can highlight the main implications of being unbanked, a complete quantitative analysis must address the general equilibrium effects of inflation. Here it is presented an extension of Dotsey and Ireland (1996) and Erosa and Ventura (2002) who construct a general equilibrium model to assess inflation costs on the economy. Moreover, the model presents a costly credit alternative to cash instead of free credit in the Lucas and Stokey (1983) framework.

4.1 Households

The main features of the previous model are retained: there are two types of households, namely H and L (i.e., banked and financially excluded). The only difference between them is the inability of agent L to acquire consumption goods through costly credit (Gillman, 1993; Dotsey and Ireland, 1996; Erosa and Ventura, 2002). The households are endowed with one unit of time, which they supply inelastically to either the good-producing labor sector (ℓ^g) or the financial labor sector (ℓ^f). They also supply capital services to firms.

Individuals draw utility from a continuum of consumptions goods, $c(j)$ where $j \in \mathcal{J} = [0, 1)$, which can be bought either with cash or with costly credit. Therefore, the household maximizes

$$\sum_{t=0}^{\infty} \beta^t u \left(\int_0^1 c_t^i(j) dj \right) \tag{8}$$

where $\beta \in (0, 1)$ and $i = \{H, L\}$. The maximization is subject to the budget constraint

$$\int_0^1 c_t^i(j) dj + \int_0^1 \xi_t^i(j) q_t(j) dj + k_{t+1}^i + \frac{m_{t+1}^i}{p_t} \leqslant w_t(\ell^{ig} + \ell^{if}) + (1 + r_t) k_t^i + \frac{m_t^i}{p_t}. \tag{9}$$

The purchase of costly credit is represented by $\int_0^1 \xi_t^i(j) q_t(j) dj$, where $\xi_t^i(j)$ is an indicator function that assumes 1 when the consumer chooses to buy good j with credit at price $q_t(j)$, and 0 if good j is bought with money. In period t, the household earns wage w_t from either sector, rental payment r_t from capital, and the amount of money accumulated from last period.

Furthermore, the household is subject to a cash-in-advance constraint:

$$\int_0^1 c_t^i(j)(1-\xi_t^i(j))dj \leqslant \frac{m_t^i}{p_t}. \tag{10}$$

and non-negativity constraints and transversality conditions for capital and money. The unbanked households are not able to smooth consumption using costly credit since they have no access to the financial sector. Then, type L agents purchase all goods using cash, that is, $\xi_t^L(j) = 0 \quad \forall t, j$.

Therefore, every period consumers choose the amount of consumption for all j, whether to buy goods using credit or cash for all j, the amount of investment in capital, and money to carry into the next period.

4.2 Firms

Given prices $q_t(j)$, w_t, the intermediary firm chooses whether to provide credit for good j, $\xi_t^s(j)$, and the amount of labor to hire, $\gamma(j)$, in order to maximize profit given by

$$\Pi_t^f = \int_0^1 \xi_t^s(j)[q_t(j) - w_t\gamma(j)]dj. \tag{11}$$

The number of workers $\gamma(j)$ increases with j in the sense that as the index of goods grows more workers are required to check the identity and credit history of the consumer. An additional assumption is $\lim_{j\to 1}\gamma(j) = \infty$, which guarantees that some goods will be bought with cash (Gillman, 1993).

Profit-maximization results in the following demand for labor:

$$n^f = \int_0^1 \xi_t^s(j)\gamma(j)dj. \tag{12}$$

The firms in the goods sectors have a constant returns to scale production function $F(K_t, n_t^g)$, where K is aggregate capital and n^g is aggregate labor in the goods sectors. Capital depreciates at rate δ. Therefore, the firms choose the production factors, given prices w_t and r_t, to maximize profit:

$$\Pi_t^g = F(K_t, n_t^g) - (r_t + \delta)K_t - w_t n_t^g. \tag{13}$$

4.3 Equilibrium

The equilibrium in this model economy must satisfy the following market clearing conditions:

$$\begin{aligned}
&N^H \int_0^1 c_t^H(j)dj + N^L \int_0^1 c_t^L(j)dj + G_t = F(K_t, L_t^g) + K_{t+1} - (1-\delta)K_t \\
&N^H k_{t+1}^H + N^L k_{t+1}^L = K_{t+1} \\
&N^H m_{t+1}^H + N^L m_{t+1}^L = m_{t+1}^s \\
&N^H \ell_t^{fH} + N^L \ell_t^{fL} = n_t^f \\
&N^H \ell_t^{gH} + N^L \ell_t^{gL} = n_t^g \\
&\xi_t^H(j) = \xi_t^s(j)
\end{aligned} \tag{14}$$

where N^i is the number of i-type agents, G_t is government expenditures, which behave as in the pure exchange model.

The competitive equilibrium for the model economy is defined as follows:

Definition 2. *A competitive equilibrium is a sequence of prices* $\{w_t, \{q_t(j)\}_{j\in\mathcal{J}}, r_t, p_t\}_{t=0}^{\infty}$*, a sequence of government policies* $\{m_0^s, \{G\}_{t=0}^{\infty}\}$*, a sequence of consumer choices*

$$\{\{c_t^i(j)\}_{j\in\mathcal{J}}, m_{t+1}^i, k_{t+1}^i, \xi_t^H\}_{t=0}^{\infty}$$

for all $i = H, L$*, a sequence of goods sector choices* $\{K_t, n_t^g\}_{t=0}^{\infty}$ *and a sequence of financial sector choices* $\{\xi_t^s(j), \gamma_t(j)\}_{t=0, j\in\mathcal{J}}^{\infty}$ *such that:*

- *Given prices and government policies, consumers maximize Eq.* (8) *subject to Eqs.* (9) *and* (10)*;*
- *Given prices and government policies, firms maximize their profits;*
- *The market clearing conditions in Eq.* (14) *hold.*

4.4 Analytical Results

This subsection shows analytically that the welfare of financially excluded individuals is lower than the welfare of individuals with access to credit when the CIA constraint binds. The next subsection quantifies these differences in welfare.

Since any consumption good j provides the same utility, one can say that for each j the consumer chooses between buying good j with money, $c_t^{H0}(j)$, and buying good j with credit, $c_t^{H1}(j)$. The following Lemma states the existence of a good such that the type-H individual is indifferent between purchasing with credit or money.

Lemma 1. *There is $s_t^H \in \mathcal{J}$ such that*

$$\xi_t^H(j) = 1, \quad c_t^H(j) = c_t^{H1} \quad \textit{for} \quad j \leqslant s_t^H \tag{15}$$

$$\xi_t^H(j) = 0, \quad c_t^H(j) = c_t^{H0} \quad \textit{for} \quad j > s_t^H. \tag{16}$$

Proof. The proof follows directly from type-H individual's first order conditions, from intermediary firms' profit maximization and equilibrium conditions. See appendix B. □

Also, from the first order conditions of both representative agents, we have the following result:

$$R_{mt} \equiv \frac{p_t}{p_{t+1}} = \frac{\lambda_t}{\beta(\lambda_{t+1} + \phi_{t+1})} < \frac{\lambda_t}{\beta\lambda_{t+1}} = (1 + r_{t+1}) \equiv R_t, \tag{17}$$

In words, the cash-in-advance constraint binds if and only if the return from capital is more significant than the rate of return on money, which implies a positive net nominal interest rate.

Proposition 1 follows from this result and the first order conditions of both representative agents:

Proposition 1. *If the net nominal interest rate is positive, then the aggregate consumption of financially excluded households is less than the aggregate consumption of households with access to credit.*

Proof. If the net nominal interest rate is positive then $\phi > 0$, which implies from Eqs. (21), (22) and (26), and $\gamma_s > 0$ that $s_t^H > 0$. Therefore, the aggregate consumption of the type-H is:

$$C_t^H \equiv \int_0^1 c_t^H(j)dj = \int_0^{s_t^H} c_t^{H1}dj + \int_{s_t^H}^1 c_t^{H0}dj = s_t^H c_t^{H1} + (1 - s_t^H)c_t^{H0} =$$
$$s_t^H u_c^{-1}(\lambda_t) + (1 - s_t^H)u_c^{-1}(\lambda_t + \phi_t)$$

while the aggregate consumption of type-L agents is

$$C_t^L \equiv \int_0^1 c_t^L(j)dj = \int_0^1 u_c^{-1}(\lambda_t + \phi_t)dj = u_c^{-1}(\lambda_t + \phi_t).$$

Then, given that $u_{cc} < 0$ implies $(u_c^{-1})'(\cdot) < 0$, $C_t^H > C_t^L$.

□

4.5 Quantitative analysis

For the quantitative analysis, final goods firms produce using a Cobb-Douglas technology

$$F(K, n_t^g) = K^\alpha (n^g)^{1-\alpha}.$$

The competitive feature of both firms assures that the wage is equal between sectors, and, therefore, the demand for labor in financial services is

$$n_t^f = \int_0^s \gamma(i)di.$$

For the consumer, the steady state choices are computed using dynamic programming. The Bellman equation for the consumer is

$$V^i(k^i, m^i, K, \pi) = \max_{c^i, s^i, (k^i)', (m^i)'} \{u(c^i) + \beta V[(k^i)', (m^i)', K', \pi']\}, \tag{18}$$

where constraints Eqs. (2) and (3) are rewritten as

$$c^i + w(K, \pi) \int_0^{s^i} \gamma(i)di + (k^i)' + (m^i)'(1 + \pi) \leqslant w(K, \pi) + (r(K, \pi) + \delta)k^i + m^i, \tag{19}$$

and

$$c^i(1 - s^i) = m^i, \tag{20}$$

where $c = \inf\{c_0, \ldots, c_1\}$ is aggregate consumption[3] and s^i is the fraction of goods paid for with credit as shown previously. Remember, therefore, that $s^L = 0$ since a financially excluded

[3] See Erosa and Ventura (2002).

individual is not able to acquire credit.

Definition 3. *A Recursive Competitive Equilibrium is a collection of household functions*

$$\{V^i, g_c^i, g_s^i, g_k^i, g_m^i\}_{i=H,L},$$

price functions $\{w, r\}$ *and aggregate laws of motion* $\{G_K, G_\pi\}$ *such that*

1. *Given prices and aggregate laws of motion,* V^i *is the solution to Eq.* (18) *subject to Eqs.* (19) *and* (20)*; and* $g_c^i, g_s^i, g_k^i, g_m^i$ *are the associated policy rules for all* $i = H, L$*, such that* $g_s^L = 0$*.*

2. *From the firm's problem:*

   (a) $r(K, \pi) = \alpha K^{\alpha-1}[n^g(K, \pi))^{1-\alpha}$

   (b) $w(K, \pi) = (1-\alpha)K^\alpha[n^g(K, \pi)]^{-\alpha}$

   (c) $n^f(K, \pi) = \int_0^{g_s^H(K,\pi)} \gamma(i)di$

3. *Government policies imply* $\pi' = G_\pi(K, \pi)$*; and*

4. *Aggregations:*

   (a) $G_K(K, \pi) = \lambda^H g_k^H(K, \pi) + \lambda^L g_k^L(K, \pi)$

   (b) $n^g(K, \pi) + n^f(K, \pi) = 1$

The welfare cost of inflation is computed in the same fashion as in the previous section. That is, the welfare cost measures how much consumption an individual requires to keep the same level of utility as in the baseline case.

The instantaneous utility function is defined as log:

$$u(c) = \log c.$$

Following Gillman (1993) and Dotsey and Ireland (1996), the transaction technology is assumed to be independent of the consumption level such that

$$\gamma(x) = \gamma \left[\frac{x}{1-x}\right]^\theta.$$

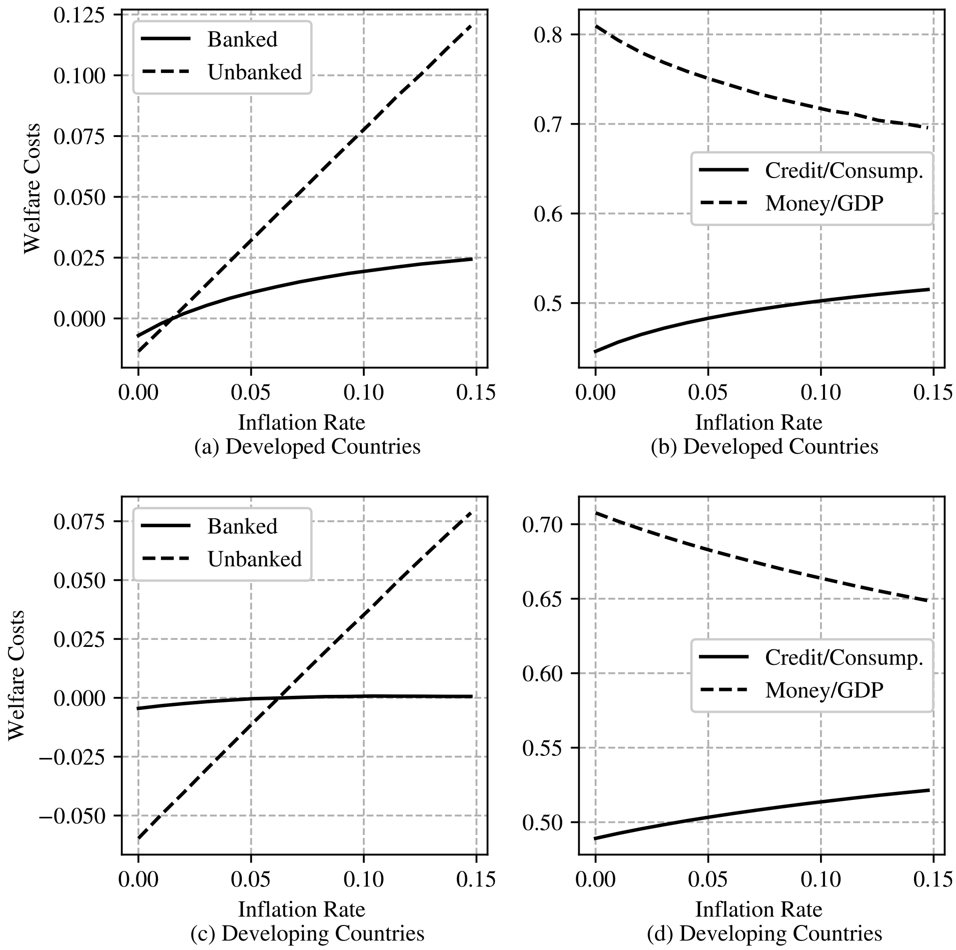


Figure 4: Inflation costs for a model with costly credit

We also need to calibrate some extra parameters. Capital's share of income, α, is set to 0.35, while the capital depreciation rate, δ, is 0.025, which are common values from the literature. Parameters γ and θ for the transaction technology are taken from Erosa and Ventura (2002) such that $\gamma = 0.0421$ and $\theta = 0.3232$. These parameters are appropriate for a model period of one quarter. The discount factor β is set equal to 0.99 for developed countries and 0.96 for developing countries. Additionally, $\lambda^H = 0.06$ for developed countries and $\lambda^H = 0.26$ for developing countries. Also, recall that the baseline annual inflation rates are 1.5% and 6.2%, respectively. The results are presented in annualized values in Fig. 4.

The qualitative implications from the model are the same as in the pure exchange economy: financially excluded individuals are more damaged by inflation than individuals with access to credit. Fig. 4a depicts these costs for developed countries. While a reduction in inflation from 1.5% to zero is equivalent to an increase of magnitude 1.36% in the consumption of underbanked and unbanked individuals, it provides a consumption equivalent of only 0.7% for banked individuals. The intuition for this result follows from the fact that financially excluded agents would benefit more from zero inflation since, without credit, they cannot protect themselves from inflation.

Note also that the quantitative result is small, but the welfare costs of inflation are higher when compared with the results in section 3. This result reflects the fact that inflation also reduces the amount of production in the economy since households must hold more cash (Dotsey and Ireland, 1996). The results, however, remain consistent with the literature in the sense that inflation imposes relatively low welfare costs on individuals in the developed economy. Also, in line with intuition, the ratio of real money to GDP declines as inflation increases, whereas the share of consumption acquired with credit increases. Fig. 4b presents these results.

For developing countries, the costs of inflation are even higher for financially excluded households relative to banked households. Fig. 4c exhibits the results. A reduction of inflation from 6.2% to zero is equivalent to an increase in consumption of 6% for unbanked individuals, while this value is only 0.44% for individuals with access to credit. Therefore, these costs can be more than 13 times higher for financially excluded households in developing countries where inflation rates are considerably higher than in developed countries.

5 Conclusion

Financially excluded households' have limited access to credit and banking services. These households are unable to smooth consumption or protect themselves from the depreciating value of their money holdings. Furthermore, financially constrained households are usually located in the lower tail of the wealth distribution, and as a consequence, inflation causes significant damage to these households.

The findings in this paper are in line with the literature that finds that the costs of inflation are relatively low in developed countries. However, the results show that financial exclusion has critical distributional effects on the welfare costs of inflation. The welfare costs are significantly higher for financially excluded households than for agents with access to credit. Furthermore, in developing economies, individuals that are financially excluded would give up more than 13 times the amount of consumption that the banked would be willing to give up in order to have a zero inflation tax.

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Appendix A: Steady States

Pure Exchange Economy

The system of equations that characterize the steady state in the pure exchange economy is:

$$
\begin{aligned}
c_1^H + c_2^H + \pi m^H &= w^H + \frac{1-\beta}{\beta} b^H \\
c_1^L + \pi m^L &= w^L + \frac{1-\beta}{\beta} b^L \\
c_1^H &= m^H \\
c_1^L &= m^L \\
\lambda^H (c_1^H + c_2^H) + \lambda^L c_1^L &= w^H + w^L \\
\lambda^H b^H + \lambda^L b^L &= 0 \\
c_2^H &= \frac{1}{\beta}(1+\pi) c_1^H
\end{aligned}
$$

where $r = 1/\beta - 1$ in the steady state.

Economy with production and costly credit

In the steady state $r = 1/\beta - \delta$ and $n^g(s^H) = 1 - \int_0^{s^H} \gamma(j) dj$. Therefore we can write aggregate capital and wages as a function of s^H such that

$$
K(s^H) = \left(\frac{\alpha\beta}{1-\delta\beta} \right)^{\frac{1}{1-\alpha}} n^g(s^H) \quad \text{and} \quad w(s^H) = (1-\alpha) \left(\frac{K(s^H)}{n^g(s^H)} \right)^{\alpha}.
$$

The system of equations that characterize the steady state is:

$$
\begin{aligned}
c^H + w(s^H)\int_0^{s^H} \gamma(j)dj + \pi m^H &= w(s^H) + \frac{1-\beta}{\beta}k^H \\
c^L + \pi m^L &= w(s^H) + \frac{1-\beta}{\beta}k^L \\
c^H(1-s^H) &= m^H \\
c^L &= m^L \\
\lambda^H(c^H + \delta k^H) + \lambda^L(c^L + \delta k^L) &= K(s^H)^\alpha n^g (s^H)^(1-\alpha) \\
\lambda^H k^H + \lambda^L k^L &= K(s^H) \\
w(s^H)\gamma(s^H) &= [(1+\pi)/\beta - 1]c^H.
\end{aligned}
$$

Appendix B: Proof of Lemma 1

This proof is similar to the one found in Dotsey and Ireland (1996). Let $\beta^t\lambda_t$ be the Lagrange multiplier on the budget constraint and $\beta^t\phi_t$ be the Lagrange multiplier on the cash-in-advance constraint. Then, the first order conditions from the type-H agent lead to:

$$c_t^{H0}(j) = u_c^{-1}(\lambda_t + \phi_t) \tag{21}$$

$$c_t^{H1}(j) = u_c^{-1}(\lambda_t) \tag{22}$$

$$\xi_t^H(j) = \begin{cases} 1 \text{ if } u(c_t^{H1}) - \lambda_t[c_t^{H1} + q_t(j)] \geqslant u(c_t^{H0}) - (\lambda_t + \phi_t)c_t^{H0} \\ 0 \text{ otherwise.} \end{cases} \tag{23}$$

Moreover, profit maximization for the firm in the intermediary sector leads to the following supply choice:

$$\xi_t^s(j) = \begin{cases} 1 \text{ if } q_t(j) \geqslant w_t\gamma(j) \\ 0 \text{ otherwise.} \end{cases} \tag{24}$$

Note that Eq. (24), together with the zero profit condition for the credit service market and the equilibrium for this market implies that

$$q_t(j) = w_t\gamma(j), \tag{25}$$

for all j demanded by type-H individuals, i.e., $\xi_t^H(j) = 1$.

Let $s_t^H \in \mathcal{J}$ be the good for which type-H individuals are indifferent between buying with credit or money, such that the inequality in Eq. (23) holds with equality. Substituting Eqs. (21), (22) and (24) into Eq. (23) at equality yields:

$$\gamma(s_t^H) = \frac{u[u_c^{-1}(\lambda_t)] - \lambda_t u_c^{-1}(\lambda_t) - u[u_c^{-1}(\lambda_t + \phi_t)] + (\lambda_t + \phi_t)u_c^{-1}(\lambda_t + \phi_t)}{\lambda_t w_t}, \tag{26}$$

which defines s_t^H.